Exam. Code : 103205
Subject Code : 9262

B.A./B.Sc. 5th Semester (Old Sylb. 2016)

MATHEMATICS

(Linear Algebra)

Paper—II

Time Allowed—3 Hours] [Maximum Marks—50

Note :— Attempt any **FIVE** questions in all, choosing at least **TWO** from each Section.

SECTION—A

I. (a) Let Q* denotes the set of all rational numbers except 1, then show that Q* forms an infinite abelian group under the operation o defined by $a \circ b = a + b - ab$ for all $a, b \in Q^*$.

(b) Define field and give an example of a ring which is not a field. 5,5

II. (a) Is R a vector space over C ? Justify.

(b) If S is any subset of a vector space V(F), then show that S is a subspace if and only if L(S) = S. 4,6

III. (a) Define complementary subspaces and give an example of subspaces that are not complementary.

(b) Find condition on a, b, c so that the vector $v = (a, b, c) \in R^3$ belongs to the space generated by $u = (1, -1, 2)$, $v = (0, 3, -4)$, $w = (2, 1, 0)$. 5,5

IV. (a) Define basis and finite dimensional vector space.

(b) Let V(R) denotes the vector space of all functions from R to R. Let $W_1 = \{f | f \in V \text{ and } f(-x) = f(x)\}$, the set of all even functions and $W_2 = \{f | f \in V \text{ and } f(-x) = -f(x)\}$, the set of all odd functions be two subspaces of V(R). Show that :

(i) $V = W_1 + W_2$

(ii) $W_1 \cap W_2 = \{0\}$

(iii) $V = W_1 \oplus W_2$. 2,8

V. (a) Extend B = {(1, 1, 1, 1), (1, 2, 1, 2)} to a basis of $R^4(R)$.

(b) Let W be a subspace of a finite dimensional vector space V(F). Then prove that :

$$\dim V/W = \dim V - \dim W.$$ 4,6

SECTION—B

VI. (a) Let V be a finite dimensional vector space and $T : V \to V$ is a linear transformation such that rank T^2 = rank T. Show that range and null space of T are disjoint.

(b) Verify rank-nullity theorem for $T : R^4 \to R^3$ defined by $T(x, y, z, w) = (x + y, y - z, z - w)$. 5,5

VII. (a) Prove that every n-dimensional vector space over field F is isomorphic to the space F^n.

(b) Give an example of a linear transformation $T : V \to V$ such that its range and null space are identical. 6,4

VIII. (a) Prove that a linear operator $T : V \to V$ is invertible if and only if T is one-one and onto.

(b) Find inverse of linear operator T on R^3 defined by $T(x, y, z) = (x - 2y - z, y - z, x)$. 6,4

IX. (a) Give an example of a linear operator $T : R^3 \to R^3$ such that $T \neq 0$ but $T^2 = 0$. Justify your assertion.

(b) Let $T : V \to V$ be a linear operator, where V is a finite dimensional vector space over field F. Suppose $B = \{v_1, v_2, ..., v_n\}$ is a basis of V(F). Prove that $[T; B] [v; B] = \{T(v); B]$ for any vector $v \in V$.

3,7

X. (a) Find linear mapping $T : R^2 \to R^3$ determined by the matrix

$$A = \begin{bmatrix} 0 & 2 \\ 1 & -1 \\ 2 & 3 \end{bmatrix}$$

with respect to ordered basis $B_1 = \{(1, 2), (0, 3)\}$ and $\{(1, 1, 0), (0, 1, 1), (1, 1, 1)\}$ for R^2 and R^3 respectively.

(b) Let $S = \{1, i\}$ and $T = \{2 + i, 1 + 3i\}$ be two basis of vector space C(R) of complex numbers over reals, what is the transition matrix from basis T to basis S ? 7,3